1. A gambler is investigating a game in a casino. He played the game 1000 times. The bet is a fixed amount of $1.00 each game. There are only three possible outcomes. Case I is he loses the bet, Case II is he wins $9 or Case III is he wins $99. From the 1000 times he played, Case I occurred 981 times, Case II occurred 15 times, and Case II occurred 4 times. Let $X$ denote the amount of money he lost or won in each game, that is, $X$ takes on $-1$, 9, or 99.

1) Find the empirical probability of losing the bet in a game.
2) Use the data he observed to estimate a probability model for the random variable $X$. That is, use the relative frequency to define the empirical probability density function of $X$.
3) Draw a probability line chart for the empirical p.d.f. of random variable $X$.
4) Graph the empirical distribution function of $X$.

2. Seven lots of components are ready to be shipped by a certain supplier. The number of defective components in each lot is as follows:

Lot 1: 3
Lot 2: 0
Lot 3: 2
Lot 4: 2
Lot 5: 1
Lot 6: 2
Lot 7: 0

One of these lots is to be randomly selected for shipment to a customer. Let $X$ denote the number of defectives in the selected lot.

6. Find the p.d.f. of $X$.
7. Graph the p.d.f. and d.f. of $X$.
8. Find the probability of having no defectives in the shipment.

3. Is $f(x) = \frac{|x|}{9}$, $x = -2, -1, 1, 2, 3, 6$, a proper probability density function? Explain your answer using the properties of probability density function.

4. Let the p.d.f. of a random variable $X$ be defined by $f(x) = \frac{x}{6}$, $x = 1, 2, 3$.
   a) $P(X > 1)$ =
   b) Find the expected value and variance of $X$.

5. If the probability density function of a continuous random variable $X$ is given by
   
   $$f(x) = \begin{cases} 
   2(1-x) & \text{for } 0 < x < 1 \\
   0 & \text{elsewhere}
   \end{cases}$$
   a) Find the expected value and the variance of $X$.
   b) Find the probability $P(X = 0.5)$.
   c) Find the distribution $P(X = 0.5)$.
   d) Find the median of this distribution.