Show all works

1. Textbook exercises: 6.1(a,c,e) 6.7(a,c,e)

2. If a sample of six values is taken from a distribution with a p.d.f. \( f(x) = 2(\theta - 1)\theta^x \),
   a) find the maximum likelihood estimator of \( \theta \).
   b) Find the estimate for \( \theta \) using the M.L.E. if the sample values are 0.27, 0.42, 0.33, 0.26, 0.11, 0.15.

3. Let \( X_1, X_2, \ldots, X_n \) be a random sample from a uniform distribution \( U(\alpha, \beta) \) with \( \alpha = 0 \). We have learned that the largest sample value \( Y_n \) is the M.L.E. for \( \beta \).
   a) Show that \( Y_n \) is a biased estimator of \( \beta \). Also, find an unbiased estimator for \( \beta \) based on \( Y_n \). (Use the p.d.f. of order statistics in page 198.)
   b) Find an unbiased estimator of \( \beta \) based on \( Y_1 \).