### Study of the effectiveness of antidepressant

<table>
<thead>
<tr>
<th></th>
<th>Relapse</th>
<th></th>
<th>Row Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Desipramine</td>
<td>14</td>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>Lithium</td>
<td>6</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>Placebo</td>
<td>4</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Column Total</td>
<td>24</td>
<td>48</td>
<td>72</td>
</tr>
</tbody>
</table>

**Hypothesis:**
- Ho: There is NO relation between variable 1 (treatment) and variable 2 (outcome variables).
- Ha: There is relation between two variables.

### Compare Observed and Expected Frequencies

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**Test Statistic:**

\[
\chi^2 = \sum_{i=1}^{r} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 (r-1)(c-1)
\]

**Cochran’s guidelines:** (Assumption: Large sample.)
- None of the expected cell counts less than 1
- No more than 20% of the expected cell frequencies are less than 5.

### Decision Rule:
If \( \chi^2 > \chi^2_\alpha \) or \( p\text{-value} < \alpha \), the null hypothesis is rejected.

Test Statistics:
\[
\chi^2 = \frac{(14-8)^2}{8} + \frac{(10-16)^2}{16} + \frac{(6-8)^2}{8} + \frac{(18-16)^2}{16} + \frac{(4-8)^2}{8} + \frac{(20-16)^2}{16} = 10.5
\]

\( \chi^2 \text{ d.f.} = (3-1)(2-1) = 2 \Rightarrow \chi^2_{.05} = 5.99 \) (Chi-square table)

C.V. approach: Since \( \chi^2 = 10.5 > \chi^2_{.05} = 5.99 \), so we reject null hypothesis. (See Chi-square Table.)

**p-value approach:** With \( \chi^2 = 10.5 > 9.210 \), the **p-value** of the test is less than 0.01, null hypothesis is rejected.

**Conclusion:** The relation between treatment and outcome variables is statistically significant.
Test for Categorical Variables

2 x 2 Contingency Table (A special case of r x c table)

Test Statistics:
\[ \chi^2 = \sum_{i=1}^{c} \frac{(O_i - E_i - 0.5)^2}{E_i} \sim \chi^2 (1), \text{ with } Yate's \ correction, \ "-0.5" \]

Example:

Is there a relationship between treatment and heart disease?
(Is there a difference in the percentages of heart disease between people who took Placebo and those who took Aspirin?)

<table>
<thead>
<tr>
<th>Group</th>
<th>Heart Disease</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes +</td>
<td>No -</td>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>Placebo</td>
<td>20 (14)</td>
<td>80 (86)</td>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Aspirin</td>
<td>15 (21)</td>
<td>135 (192)</td>
<td></td>
<td>150</td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
<td>215</td>
<td></td>
<td>250</td>
</tr>
</tbody>
</table>

\[ 35 \times 100/250 = 14, \ 35 \times 150/250 = 21, \ 215 \times 100/250 = 86, \ 215 \times 150/250 = 192 \]

Test Statistic:
\[ \chi^2 = \frac{(|20 - 14| - .5)^2}{14} + \frac{(|80 - 86| - .5)^2}{86} + \frac{(|15 - 21| - .5)^2}{21} + \frac{(|135 - 192| - .5)^2}{192} = 4.19 \]

\[ \text{d.f.} = (2-1)(2-1) = 1 \Rightarrow \chi^2_{0.05} = 3.84 \] (Chi-square table)

C.V. approach:
Since \( \chi^2 = 4.19 > \chi^2_{0.05} = 3.84 \), reject null hypothesis.

p-value approach:
With \( \chi^2 = 4.19, \ .025 < p\text{-value} < .05 \), null hypothesis is rejected.

Conclusion: There is significant association between the use of Aspirin and heart disease.

(For small sample, some cell counts are less than 5, Fisher's Exact Test can be used for 2x2 contingency table.)

Example: Suppose we want to determine if people with a rare brain tumor are more likely to have been exposed to benzene than people without a brain tumor. One experimental design used to answer this question. First, we start with cases, people with a disease or condition (brain tumor) and find people who are as similar as possible but who do not have brain tumors. Those people are called controls.

<table>
<thead>
<tr>
<th>Exposure</th>
<th>Case</th>
<th>Control</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>50</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td>No</td>
<td>100</td>
<td>130</td>
<td>230</td>
</tr>
<tr>
<td>Total</td>
<td>150</td>
<td>150</td>
<td>300</td>
</tr>
</tbody>
</table>

At the level of significance \( \alpha = 0.05 \), are “exposure to benzene” and “have brain tumors” independent?
Test for Categorical Variables

Testing Distribution

Chi-square Goodness of Fit Test

Test Statistics:

\[ \chi^2 = \sum_{i=1}^{c} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 (c-1) \]
degrees of freedom = \( c - 1 \) (number of intervals minus 1)

Example: The following is a random sample of size 20. Was this sample obtained from a normally distributed population?

16.7  18.8  24.0  35.1  39.8
17.4  19.3  24.7  35.8  42.1
18.1  22.4  25.9  36.5  43.2
18.2  22.5  27.0  37.6  46.2

(sample mean = 28.565, standard deviation = 9.8887)

Ho: This random sample represents observations on a “normally distributed” random variable with mean 30 and variance 100.

Ha: Ho is not true.

If Ho is true then the three quartile of this normally distributed random variables are

\( z_{.25} = 30 + 10(-.6745) = 23.255 \)
\( z_{.50} = 30 \)
\( z_{.75} = 36.745 \)

\((-\infty, 23.255] \)
\((23.255, 30]\)
\((30, 36.745]\)
\((36.745, \infty]\)

\begin{array}{ccccc|c}
\text{Observed Frequencies} & (\infty, 23.255] & (23.255, 30] & (30, 36.745] & (36.745, \infty] & \text{Totals} \\
5 & 8 & 4 & 3 & 5 & 20 \\
5 & 5 & 5 & 5 & 5 & 20 \\
\end{array}

\[ \chi^2 = \frac{(8-5)^2}{5} + \frac{(4-5)^2}{5} + \frac{(3-5)^2}{5} + \frac{(5-5)^2}{5} \]
\[ = 2.8 < 7.815 = \chi^2_{.05} (3) \]

⇒ reject Ho

⇒ The sample suggests that there is no evidence indicating that the sample is not from normal population.

What if the null hypothesis is that “Ho: This random sample represents observations from an uniformly distributed random variable between 10 and 50?”
If parameters were estimated than the test would be

\[
\chi^2 = \sum_{i=1}^{c} \frac{(O_i - E_i)^2}{E_i} \sim \chi^2 (c-1- m)
\]

Degrees of freedom = \(c - 1 - m\)

\(= \text{(number of intervals minus 1, and minus m which is the number of parameters estimated)}\)

**Example:** Number of air plans landing at a particular airport per 10 minutes at certain time of a day is recorded as following:

<table>
<thead>
<tr>
<th>Number of air plans:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>at least 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Frequency:</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>Expected Frequency:</td>
<td>5.88</td>
<td>12.34</td>
<td>12.96</td>
<td>9.07</td>
<td>7.75</td>
</tr>
</tbody>
</table>

\(48 \left[ e^{-2.1}\right] /0! = 5.88\) (rest of the expected frequencies can be calculated in the same fashion)

The average number of air plans observed per 10 minutes according the data above is 2.1, (reported by the researcher) estimate of \(\lambda\). If the last category is more specific than one can calculate the mean.

\[
\chi^2 = \frac{(9-5.88)^2}{5.88} + \ldots + \frac{(6-7.75)^2}{7.75} = 6.31 < 7.815 = \chi^2_{0.05} (3)
\]

The degree of freedom is 3 because there is one parameter estimated.

Conclusion: Failed to reject Ho. There is no sufficient evidence to say that the landing pattern of air plans doesn’t follow Poisson distribution.

*The chi-square tests above are all using limiting distributions that are supported by large sample. The **expected cell counts** should all be greater than 5 for these chi-square goodness of fit tests.*